COMPARATIVE ANALYSIS OF LOSS ALLOCATION METHODS FOR DISTRIBUTION SYSTEMS WITH DISPERSED GENERATION

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INTRODUCTION

According to structural changes in power sector, competition was introduced in generation and supply segments of electric energy business. Transmission and distribution segments of power sector are immune to competition because there is no economical and technical logic to introduce it in them. But these segments are crucial for fair competition in generation and supply. The main issue for fair competition is open access on non-discriminatory basis to transmission and distribution networks and setting adequate price for network services and operation costs. One of the main operating cost of each network are power losses and there appropriate and fair allocation is very important for efficient operation of the network and affects future sitting of generators and loads and network development. These facts rise the importance of loss allocation problem and makes it one of the hottest topics in scientific and expert interest and research.

In the past loss allocation problem was addressed only to transmission networks but with increased penetration of dispersed generation (DG) and introduction of competition for suppliers, this problem becomes very interesting for distribution networks. DG has significant technical and economic impact on distribution networks because it alters the power flows in the network from unidirectional to bidirectional and by that DG changes the losses in the network as one of the main operating costs in the network. Competition for suppliers and there interactions with consumers leads to a greater importance of economic aspects of distribution management system.

Loss allocation problem has a pure economic nature but it has to be solved with mathematical algorithm. Main difficulty in solving this problem is that branch power losses are nonlinear function of bus power injection. Authors in [1] concluded that regardless of the approach the final allocation always contains a degree of arbitrariness, because system losses are non-separable, nonlinear quadratic function of the bus power injection which makes it impossible to divide losses into the sum of terms, each one uniquely attributable to generation or load. According to this remarks, evaluation requirements have been established for comparison of different approaches for loss allocation proposed in literature. These requirements can be summarized as [2]: 1) *Economic efficiency:* Losses must be allocated in a way to reflect the true cost that each user imposes on the network; 2) *Accuracy, consistency and equity:* Loss allocation method must be accurate and equitable i.e. must avoid or minimize cross subsidies between users and between different time of use; 3) *Utilization of metered data:* From a practical standpoint it is desirable to base allocation of losses on actual metered data; 4) *Simplicity of implementation:* For any proposed method to find favour it is important that the method is easy to understand and implement.

Generally methods for loss allocation can be divided by two criteria's, one by type of network, method is dedicated to: transmission or distribution and the other one by the approach which method is based on: marginal, average and actual.

Main idea of this paper is to give critical evaluation and comparative analysis of three already proposed methods for loss allocation in distribution systems with DG. These methods are: marginal loss coefficients (MLC) [2], Z-bus method [1] and succinct method (SMLA) [3]. Selection of methods is performed according to the approach on which method is based on and on citation basis in world power system journals. Due to the approach MLC method is representative of marginal approach, Z-bus and SMLA are representatives of actual approach. Critical evaluation and comparative analysis will be done according to the efficiency of these methods when they are implemented on distribution network. Evaluation and comparison is performed on small specific test distribution network, according to three aspects: slack node treatment and its loss allocation, P and Q sensitivity of loss allocation methods, and network representation and matrix singularities.

MARGINAL LOSS COEFFICIENTS

By definition MLCs measure the change in total active power losses L due to a marginal change in consumption/generation of active power P_i and reactive power Q_i at each node *i* in the network.

$$\tilde{\rho}_{Pi} = \frac{\partial L}{\partial P_i} \quad \tilde{\rho}_{Qi} = \frac{\partial L}{\partial Q_i} \tag{1}$$

where $\tilde{\rho}_{Pi}$ and $\tilde{\rho}_{Qi}$ represent the active and reactive power related MLCs. If a user, i.e. generator takes part in voltage control by injecting required reactive power (PV node), there are no loss-related charges for the reactive power to be allocated: $\tilde{\rho}_{Qi} = \frac{\partial L}{\partial Q_i} \overset{def}{=} 0$ *i* is a PV node. Since in load flow calculations, losses are deemed to be supplied from the slack node, the loss-related charges for this node are zero. In other words, total power losses are insensitive to changes in active and reactive injections at the slack node i.e. $\frac{\partial L}{\partial P_s} = \frac{\partial L}{\partial Q_s} = 0$, s is the slack node.

MLCs are a function of a particular system operating point. As there is no explicit relationship between losses and power injections the standard chain rule is applied in the calculations of MLCs using intermediate state variables, voltage magnitudes and angles. Therefore only a load flow solution for a particular system operating point is required to compute MLCs.

Applying the standard chain rule, the following general system of linear equations can be established for calculating MLCs:

$$\mathbf{A} \cdot \tilde{\boldsymbol{\rho}} = \mathbf{b} \tag{2}$$

Matrix A is the transpose of the Jacobian in the Newton-Raphson load flow and can be calculated on the basis of load flow results for a particular system operating point. The vector $\tilde{\boldsymbol{\rho}}$ represents MLCs whereas the right-hand vector **b** represents sensitivities of total losses with respect to voltage angle and magnitude (θ, U). Total system active loss *L* is given by:

$$L = \sum_{i=1}^{N} \sum_{j=1}^{N} G_{ij} \left[U_i^2 + U_j^2 - 2U_i U_j \cos(\theta_i - \theta_j) \right] \tag{3}$$

Therefore the entries of vector b in eqn. (2) are:

$$\frac{\partial L}{\partial \theta_i} = 2 \sum_{j=1}^{N} G_{i-j} U_i U_j \sin(\theta_i - \theta_j) \qquad i = 1, \ldots N \tag{4}$$

$$\frac{\partial L}{\partial U_i} = 2\sum_{j=1}^{N} G_{i-j}\left[U_i - U_j\cos(\theta_i - \theta_j)\right] \quad i = 1,....N \tag{5}$$

Note that there are no equations for any voltage-controlled node as by definition the MLC with respect to reactive power for any such node is zero. The result of applying MLCs calculated in accordance with the procedure outlined yields approximately twice the amount of losses. That is:

$$\sum_{i=1}^{N-1}\left[\widetilde{\rho}_{Pi} \cdot P_i + \widetilde{\rho}_{Qi} \cdot Q_i\right] \approx 2 \cdot L \tag{6}$$

Therefore there is a need of reconciliation. Constant multiplier reconciliation factor k_0 is introduced in order to obtain vector of reconciled MLCs ρ. The factor k_0 is calculated as follows:

$$k_0 = \frac{L}{\sum_{i=1}^{N-1}\left[\widetilde{\rho}_{Pi} \cdot P_i + \widetilde{\rho}_{Qi} \cdot Q_i\right]} \tag{7}$$

The vector of reconciled MLCs ρ is then calculated as follows:

$$\rho = k_0 \cdot \widetilde{\rho} \tag{8}$$

Reconciled MLCs enable the allocation of the total system active power losses to individual users such that:

$$\sum_{i=1}^{N-1} \rho_{P_i} \cdot P_i + \sum_{i=1}^{N-1} \rho_{Q_i} \cdot Q_i = L \tag{9}$$

Z-BUS AND SUCCINCT METHODS

Common characteristic of these two methods is the actual approach which they use for loss allocation. Both methods are circuit based, using the system structure, expressed by bus impedance matrix $\mathbf{Z_{bus}}$ and on results of a power flow calculation. For a distribution system with N+1 nodes (slack at node 0) and B branches, the components of the matrix $\mathbf{Z_{bus}}$ are denoted as $\underline{Z}_{ik} = R_{ik} + \mathrm{j}X_{ik}$, for *i*=0,1,…,N, *k*=0,1,…,N. The power flow solution provides the complex node voltage $\underline{U}_k$, the node injected complex power $\underline{S}_k = P_k + jQ_k$, and the node injected current $\underline{I}_k$. Allocation of losses with Z-bus method is defined with equation (10), where L_k are allocated losses to node *k*:

$$L_k = \mathrm{Re}\left[\underline{I}_k^*\left(\sum_{m=0}^{N} R_{km} \cdot \underline{I}_m\right)\right] \tag{10}$$

Succinct method for loss allocation (SMLA) is developed for allocating only the variable loss due to the series impedance branch, whereas the invariable loss due to the shunt admittance branch must be allocated in average terms among all network users. Let us consider *i* and *j* as the sending and receiving nodes of branch *l*, and let us assume $\underline{Z}^{(l)}$ to be the impedance of branch *l*, for l=1,……, B. The losses L_k allocated to each node k=0,……..N are:

$$L_k = \mathrm{Re}\left\{ \sum_{l=1}^{B} \frac{\left(\underline{Z}_{ik} - \underline{Z}_{jk}\right)}{\underline{Z}^{(l)}} \left(\frac{\underline{U}_i - \underline{U}_j}{\underline{U}_k} \right)^* \left(P_k - jQ_k\right) \right\} \tag{11}$$

COMPARATIVE ANALYSIS OF THE METHODS

Comparative analysis between elaborated methods will be performed according to three aspects in order to evaluate methods applicability to distribution networks and to investigate their efficiency for loss allocation due to the requirements for achieving ideal loss allocation. These aspects are: slack node loss allocation, P and Q sensitivity of the loss allocation methods, and matrix singularities.

Slack node loss allocation

For better evaluation of slack node impact on loss allocation with elaborated methods, a simple test distribution system (see Fig. 1) will be used. It is consisted of a single feeder (cable line) supplying 10 identical loads. The base voltage is 10 kV, and the base power is 1 MVA. The feeder portions between two nodes are with equal length and following parameters: resistance $R = 2.06 \cdot 10^{-3}$ p.u., reactance $X = 0.8 \cdot 10^{-3}$ p.u. and susceptance $B = 15.7 \cdot 10^{-3}$ p.u. Loads are modelled with constant power model with $P = 0.2$ p.u. and $Q = 0.1$ p.u.

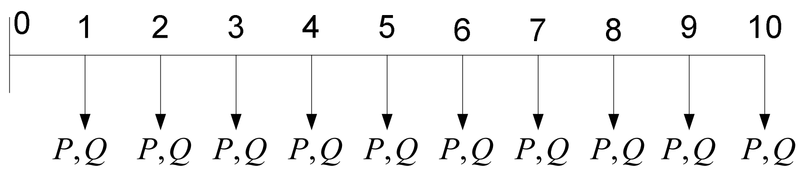


Fig. 1. Simple test distribution system

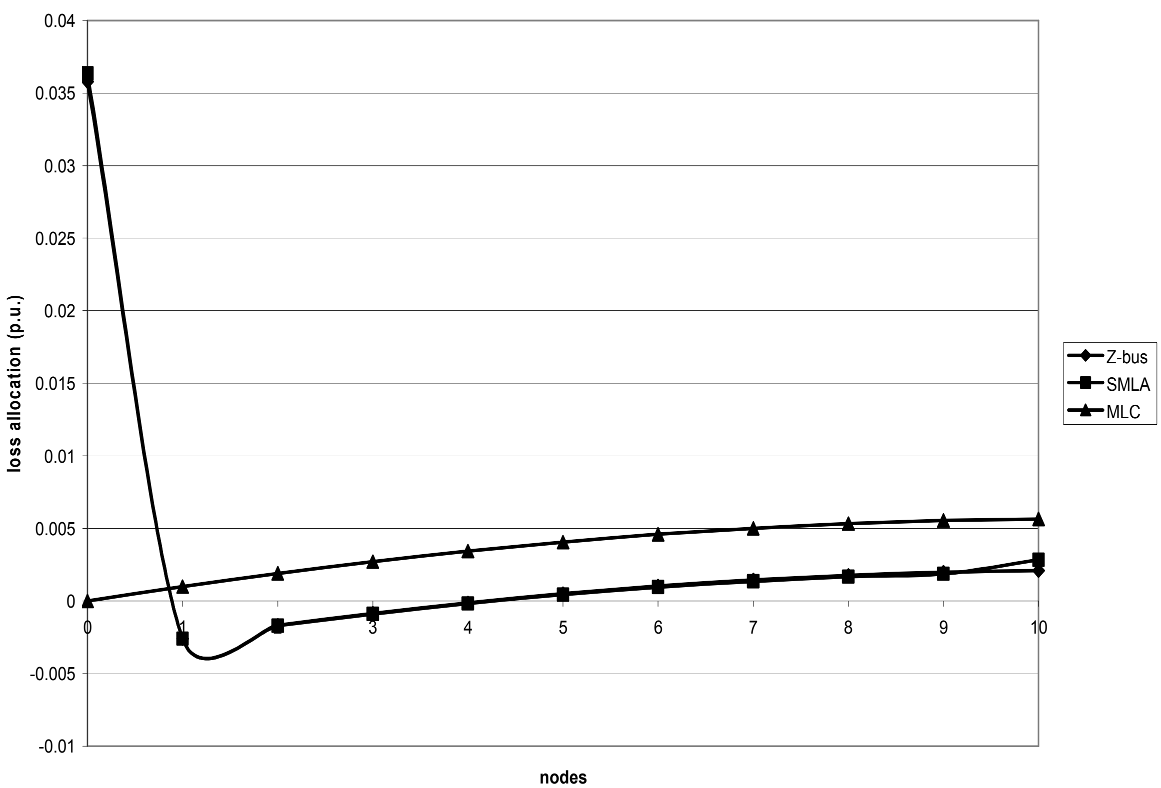


Fig. 2. Allocation of losses to nodes of test distribution system from Figure 1.

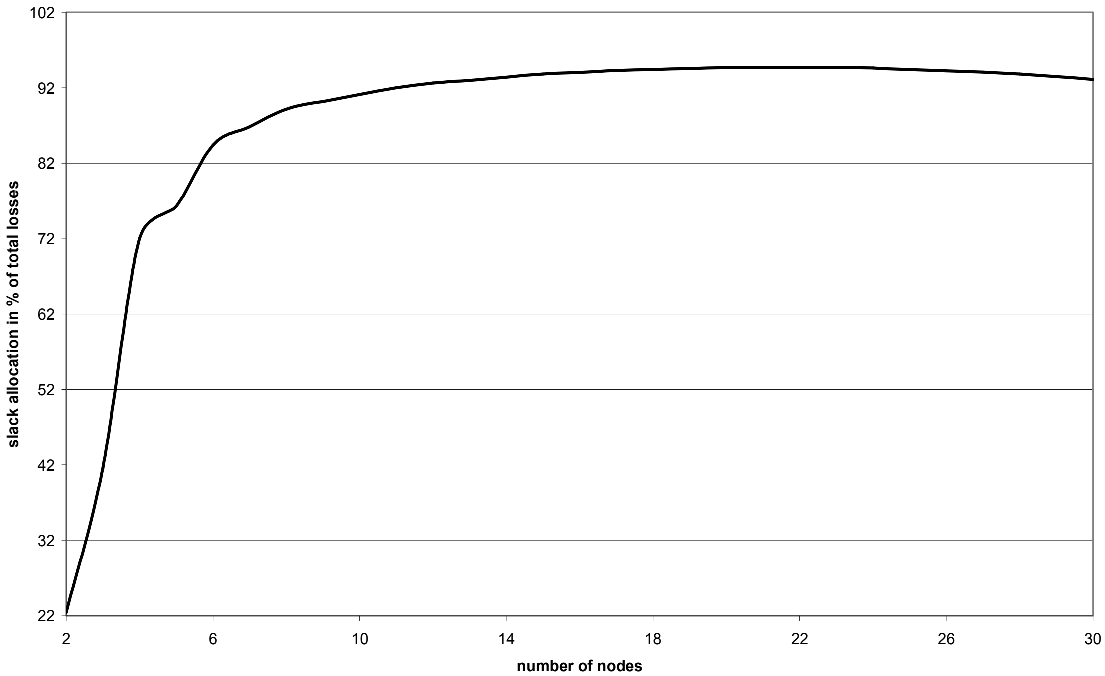


Fig. 3. Percent of total network losses allocated to the slack node with Z-bus and SMLA methods

One of the key differences between the loss allocation methods is based on the treatment of the slack node. MLC method according to its definition allocates 0 losses to the slack node.

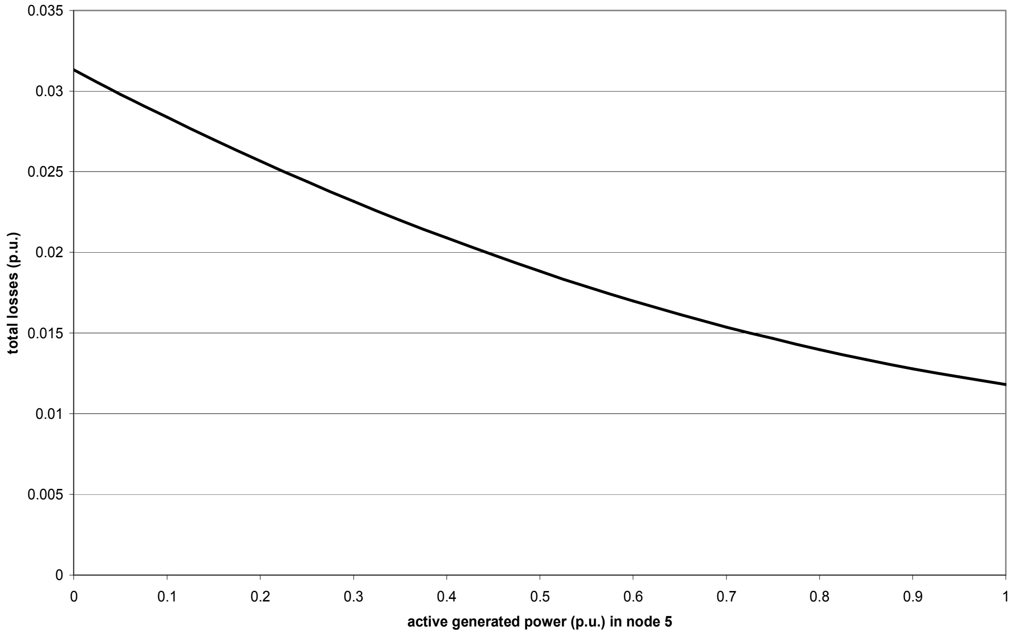
Fig. 2 shows that the Z-bus and SMLA method allocate a significant part of the losses to the slack node. It is obvious from Fig. 2 that the slack node 0 is allocated with 91% of total network losses with Z-bus and SMLA method. This is a great obstacle for implementation of these two methods in distribution system. Treatment of the slack node in the loss allocation process is still subject of many discussions. Taking into account physical and financial unbundling between transmission and distribution network, it is logical slack node to has zero allocation of losses. On the other side distribution network receives or sends electric energy from/to transmission network, and it is charged with transmission allocation of losses for the appropriate node which is a slack for distribution network. That means that same node can not be again charged for losses at distribution level. From power flow theory is well known that slack node is mathematical artefact which purpose is simulation of regulation power plant in the system. Some authors are solving this problem with implementation of distributed slack bus. This is derived from the fact that in real power systems all power plants participate in covering losses in the network. These considerations are relevant for transmission networks, but with increased penetration of DG can be applicable also for distribution networks.

Fig. 3 highlights that the portion of losses allocated to the slack node increases with the number of loads supplied by the feeder. From this figure, it is obvious that with increasing of number of nodes, slack node loss allocation reaches saturation of 93 % of total network losses.

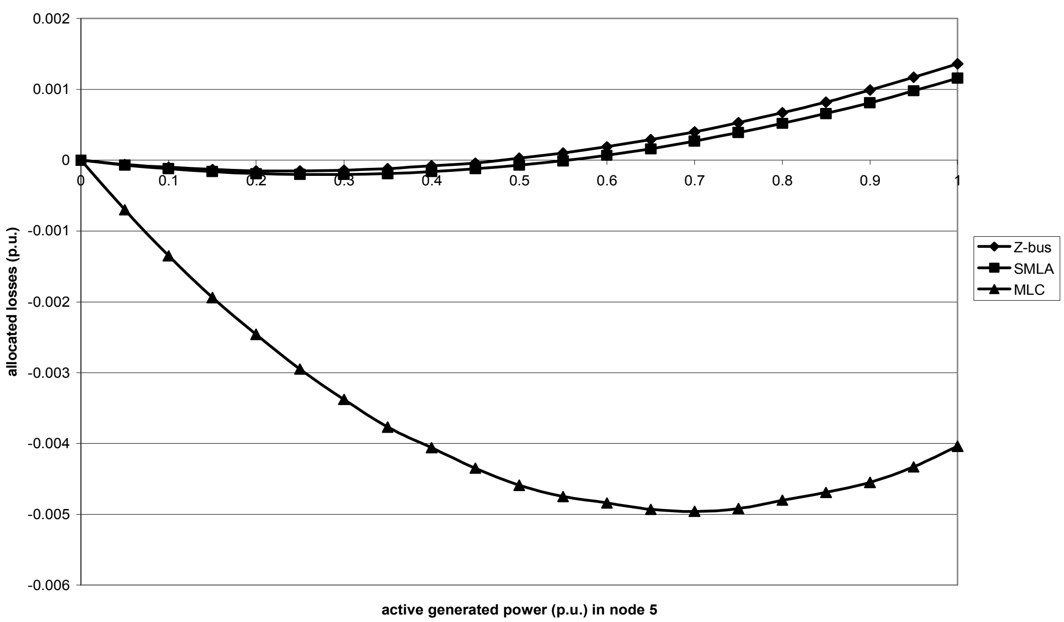
P and Q sensitivity of loss allocation methods

Loss allocation factors can be used as sensitivity factors which indicate the change in total real power losses in the network L, due to a variation in the real and reactive generation/consumption at each node k of the system. In order to investigate this sensitivity P and Q concept, it necessary to calculate loss allocation results in the presence of variable generation of P and Q at some node of the network. For that purpose at test distribution feeder shown on Fig. 1, load at node 5 is substituted with generator, first with variable active power and after that with variable reactive power. Fig. 4 shows the variation of total network losses (a) and losses allocated to node 5 (b) due to variation of active generated power at node 5.

It is obvious that with increasing of active generated power at node, total network losses are decreasing. Allocation of losses with Z-bus and SMLA is very similar, node 5 has negative loss allocation in interval of 0-0.5 kW, and after that they give positive loss allocation. MLC method results with negative loss allocation to node 5 during the hall interval of variation of active generated power at node 5.



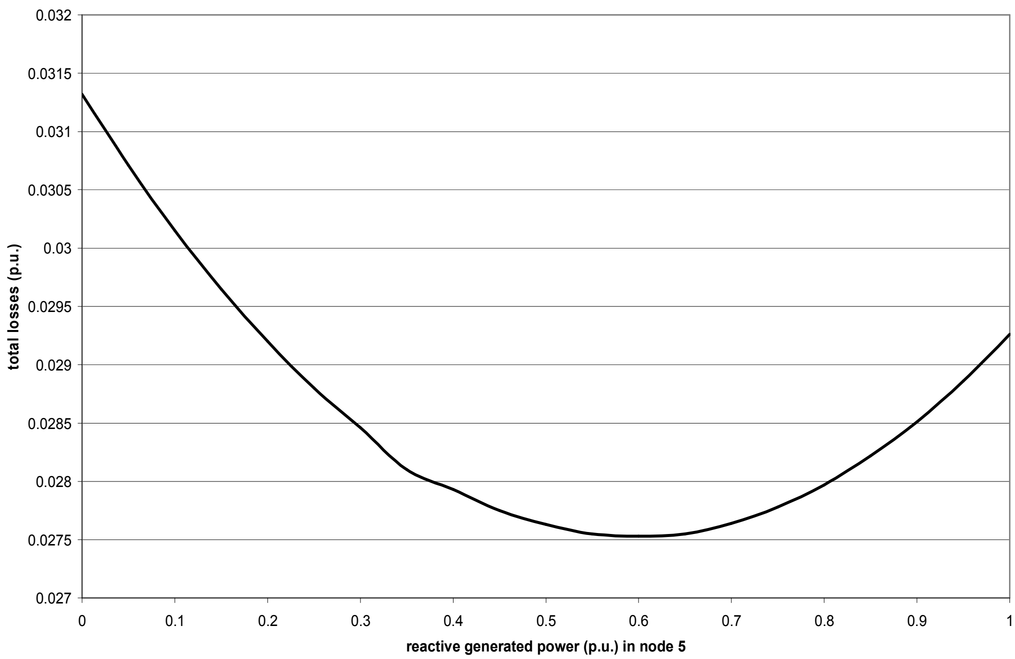
a)



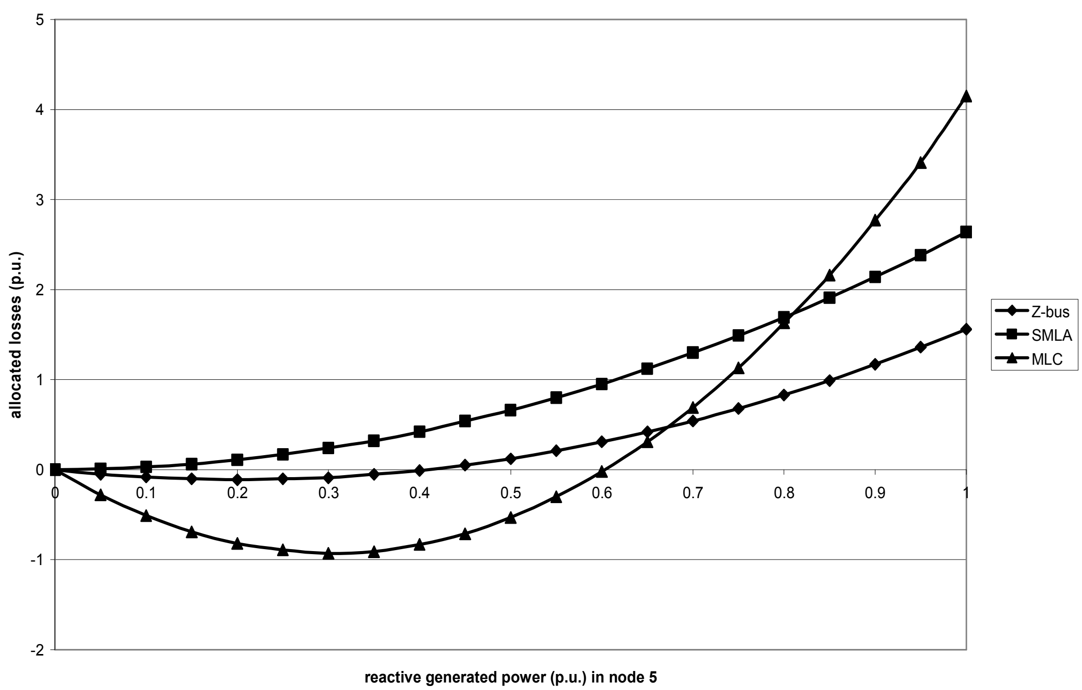
b)

Fig. 4. a). Total network losses variation due to variation of active generated power at node 5; b) Losses allocated to node 5 with Z-bus, SMLA, and MLC

Fig. 5 shows the variation of total network losses (a) and losses allocated to node 5 (b) due to variation of reactive generated power at node 5. It is obvious that with increasing of reactive generated power at node 5, total network losses are decreasing to its minimal value which corresponds to reactive generated power 0.6 p.u., and after that total network losses start to increase.



a)



b)

Fig. 5. a). Total network losses variation due to variation of reactive generated power at node 5; b) Losses allocated to node 5 with Z-bus, SMLA, and MLC

Allocation of losses with SMLA at node 5 has positive value in interval of variation of reactive generated power. This result does not correspond to the variation of total network losses and gives information that SMLA method is not efficient for reactive loss allocation coefficients. This paradox is elaborated from authors in [4]. It is shown in [4] that when $tg\varphi_{line} > tg\varphi_{load}$ reactive loss allocation coefficient obtained with SMLA has always positive value. Opposite when $tg\varphi_{line} \leq tg\varphi_{load}$ reactive loss allocation coefficient of SMLA changes its sign. MLC method results with loss allocation to node 5 which totally corresponds to total network losses variation due to reactive generated power at node 5.

Network representation and Matrix singularities

According to mathematical algorithms of these three methods it is obvious that they are based on matrix approach. This approach is not suitable for solving problems in distribution systems because of their radial or weakly meshed structure.

For the application of Z-bus method, it is important to know the bus impedance matrix, which is defined only if the admittance matrix is non-singular. It is well known that Z-bus matrix can be calculated with special algorithm for that purpose but the easiest way for its calculation is through inversion of bus admittance matrix. For distribution systems composed of overhead lines only, the shunt admittance can be negligible, so that the bus admittance matrix becomes singular, and the bus impedance matrix is not defined. The Z-bus method cannot be applied in this case. In addition, it may be very sensitive to the variation of its parameters in systems where the shunt capacitances are relatively low.

The SMLA method is able to work in absence of shunt parameters, since it does not contain directly the bus impedance matrix coefficients, but the differences among its coefficients, corresponding to indeterminate forms that can be always defined as limit cases.

Derivative based methods as MLC require the computation of bus admittance matrix and the power flow Jacobian matrix with a more computationally intensive procedure.

However the power flow calculation for distribution systems (with or without DG) can be effectively performed by using the well known backward/forward summation (current, power, admittance) methods, which do not require computing of power flow Jacobian not the bus impedance matrix. It can be concluded that for loss allocation in distribution systems with or without DG branch based approach has many advantages versus matrix approach according to specific topological structure of distribution networks.

CONCLUSION

This paper provided comparative analysis of three methods for allocation of losses in distribution system with DG. From the analysis it can be concluded that SMLA method is not able to provide efficient loss allocation under specific circumstances concerning the reactive power load. Z-bus and SMLA methods allocate a significant part of the losses to the slack node, and Z-bus method is very sensitive to to the variation of its parameters in systems where the shunt capacitances are relatively low. MLC method require the computation of bus admittance matrix and the power flow Jacobian matrix with a more computationally intensive procedure. It can be concluded that for loss allocation in distribution systems with or without DG branch based approach has many advantages versus matrix approach according to specific topological structure of distribution networks.

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